Math 1A, Fall 2010 — M. Christ
First Midterm Practice Exam

(1a) Use limit rules to evaluate $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$.

(1b) Let $f(x) = \sqrt{x}$, with its natural domain. Does $f'(9)$ exist? Justify your answer.

(2a) Let $f(x) = \frac{(x-2)(x-4)(x-8)}{3(x-1)(x-3)(x-8)}$, with its natural domain. Find all asymptotes of the graph of f.

(2b) Find $\lim_{x\to 0} x^2 \sin(e^{1/x})$. Justify your answer, using methods taught in this course.

(3) Show that there is at least one real number x which satisfies $x^6 = 1 + \sin(x)$.

(4a) Let $r > 0$. How is $\log_r(2)$ defined?

(4b) Let $f(x) = \tan(x)$ with domain $(-\frac{\pi}{2}, 0)$. Does f have an inverse? If so, what are the domain and range of the inverse function?

(4c) If some *vertical* line intersects a curve at more than one point, what does this say about the curve?

(4d) Simplify: $\ln(5e\sqrt{x})$, assuming that $x > 0$.

(4e) If the domain of f contains $(-1, 1)$, and if f is continuous at 0, must $f'(0)$ exist? Either explain in words why it must exist, or give an example of a function for which it does not exist.

(5a) Let $f(x) = x^2$. Find $\delta > 0$ such that $|f(x) - 36| < \frac{1}{1000}$ whenever $|x - 6| < \delta$.

(5b) Show, using the definition of a limit, that

$$\lim_{x\to\frac{1}{3}} (9x - \frac{1}{x}) = 0.$$

Solutions to all problems will be posted, **but** I recommend that you first attempt to work the problems, before peeking at the official solutions. Compare your work with mine. In case of large discrepancies, consult your GSI.